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Prepared by
CASE INSTITUTE OF TECHNOLOGY
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A CRITICAL EVALUATION OF THE STATUS AND TRENDS IN HIGH SPEED FLUID FILM LUBRICATION

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ABSTRACT

High speed lubrication (also denoted as turbulent and super-laminar lubrication) and relevant parts of the fluid mechanics literature are reviewed. It is contended on the basis of this review that existing turbulent journal bearing calculations are predicated on an unrealistic characterization of the lubricant film. Furthermore, an order-of-magnitude argument is presented which indicates that the neglect of inertial terms in the governing equations of turbulent lubrication is incorrect. It is concluded that fundamental investigations on the structure of high speed lubricant films must precede the formulation of an adequate theory, and the nature of these investigations is discussed.

INTRODUCTION

Classical lubrication theory is based on two fundamental assumptions. Firstly, the inertial terms in the equations of motion for the lubricant film are negligible compared to the viscous terms, and secondly, lubricant films are so thin that the lateral derivatives in the governing equations are small compared to the normal derivatives. Formal statements equivalent to these two assumptions are that both the modified or lubrication Reynolds number and the ratio of the

characteristic thickness of the film to the characteristic extent of the film are small ⁽¹⁾.

In the past fifteen years applications have arisen in which the modified Reynolds number is of unit order or larger, and classical lubrication theory has proved inadequate. The larger modified Reynolds numbers are a result of the trend toward higher rotational speeds and the use of liquid metals and other fluids with low kinematic viscosities as lubricants. In the following, the term "high-speed" will be used to connote all situations in which the modified Reynolds number is of unit order or larger.

The initial investigations of high-speed lubrication were essentially of an experimental nature ^(2,3,4,5). It was shown that bearing performance differed appreciably from the predictions of classical lubrication theory when the modified Reynolds number exceeded a critical value near unity. The existence of a critical modified Reynolds number was noted in most cases as being in agreement with the earlier findings of G. I. Taylor ^(6,7) in studying the flow between concentric, rotating cylinders. It is pertinent to note here that the critical speed in Taylor's work was for the onset of a secondary flow regime. Shortly after the existence of a high-speed lubrication regime had been demonstrated experimentally, theoretical analyses of high-speed journal bearings appeared ^(8,9) in which the flow was assumed to be fully turbulent. These analyses were based on equations of motion which differed from those of classical lubrication theory by the addition of turbulent stresses. The turbulent stresses were treated

by means of the Prandtl mixing length theory in ref. 9, and a power law profile was introduced in ref. 8. Analyses similar to ref. 9 were presented shortly afterward for the infinite slider⁽¹⁰⁾ and journal bearings with end leakage^(11,12). A significant amount of disparity still was noted between the predictions of the turbulent journal bearing analyses and the data of ref. 3, and more complicated treatments for the turbulent stresses were soon advocated^(13,14,15) to improve the situation.

The present paper is devoted to a critical study of high-speed lubrication theory as of early 1966. Essentially, three questions are to be considered. Firstly, is the flow in a high-speed lubricant film likely to be turbulent? After all, the widely referenced work of Taylor concerns the onset of a secondary flow and not turbulence. The discussion of the turbulence assumption will be based on the information available on flow between rotating cylinders and will be most relevant to journal bearing configurations. Secondly, if turbulence does exist in high-speed lubricant films, are the turbulence representations of refs. 13, 14, and 15 appropriate to high-speed lubrication theory? The discussion of the turbulence representations will be based on the physical implications of the turbulence models being advocated, and the major portion of this discussion will also be related to journal bearings. Finally, under what conditions may the inertial terms in the momentum equations for the film be neglected? This has been assumed in most all existing turbulent analyses and is, therefore, a characteristic feature of the theory as it now stands. The discussion of the

inertial terms will be based on general order-of-magnitude arguments and will not be limited to a particular bearing configuration.

The works of Woodhead and Kettleborough⁽⁴⁰⁾ and Kettleborough⁽⁴¹⁾ are exceptions. Kettleborough⁽⁴¹⁾ analyzed the slider bearing with inertia, turbulent and viscous terms included. When inertia only was considered the results were in qualitative agreement with published (turbulent-attributed) slider bearing experiments. Kettleborough, therefore, concluded that the turbulent term did not significantly influence the slider bearing performance.

HIGH-SPEED JOURNAL BEARINGS AND ROTATING CYLINDER FLOWS

It has already been stated that the earliest investigations of high-speed lubrication phenomena related their results to the earlier work of Taylor^(6,7). The reason for doing so was that earliest information on the high-speed regime was obtained from journal bearing tests. Since the simplest journal bearing configuration is a pair of circular cylinders, it was thought that the flow between a rotating inner cylinder and a fixed outer cylinder should be similar to journal bearing flows. If one accepts this similarity and recognizes that Taylor demonstrated the onset of a secondary flow in this situation, it then seems odd that the ensuing journal bearing analyses^(8,9) postulated a turbulent flow regime. With this in mind, a brief review of the rotating cylinder literature will be made to seek the basis for the turbulence assumption.

Perhaps it is best to start the review with Taylor's original

work on concentric rotating cylinders⁽⁶⁾. Taylor was primarily concerned with the stability of rotating Couette flow, and his investigation consisted of a theoretical determination of the neutrally stable modes by means of small disturbance theory and an experimental confirmation of the analysis by means of flow visualization. His analysis predicted that the departure from rotating Couette flow would occur in the form of torroidal vortices for the case of rotating inner cylinder and fixed outer cylinder and that the onset of the vortices would occur when

$$\frac{U_{\theta} C}{\nu} = \frac{41.1}{\sqrt{c/r}} \quad \neq$$

for cylinders whose separation distance is small compared to their radii. In this result U_{θ} is the speed of the inner cylinder, C is the separation distance between the cylinders, r is an average radius of the cylinders, and ν is the kinematic viscosity of the fluid. (Taylor also considered the case with rotating outer cylinder and fixed inner cylinder. However, this case has little relevance to lubrication situations.)

Since Taylor's original contribution, additional experimental studies have been reported by Taylor⁽⁷⁾, Pai⁽¹⁶⁾, Schultz-Grunow and Hein⁽¹⁷⁾, Nissan et al.⁽¹⁸⁾, Coles⁽¹⁹⁾ and Donnelly and Schwarz⁽²⁰⁾. All of these investigations indicate a transition from Couette flow to a secondary vortex regime at the predicted critical speed. The more

[≠]This form of Taylor's result is due to L. Prandtl.

recent studies also indicate that the vortices become wavy, i.e., a circumferential disturbance sets in, as the rotational speed is increased beyond the critical. However, both the required increase in speed for the onset of the circumferential modes and the number of circumferential waves at the onset appear to be functions of the individual apparatus. As the speed of rotation is further increased, the circumferential wave number increases until the flow can be described as having a continuous spectrum of non-axisymmetric frequencies⁽¹⁹⁾ or having turbulence superposed on a basic torroidal vortex structure^(16,18). However, the vortex structure appears to be a consistent feature of the flow at all velocities greater than the Taylor critical speed.

In addition to experimental work, there has been a considerable amount of analytical interest in the flow development after the onset of the Taylor vortices. Stuart⁽²¹⁾ and Davey⁽²²⁾ have calculated the increase in vortex strength as the rotational speed is increased past its critical Taylor value. Their predictions of frictional torque are in good agreement with the experimental results obtained on both a rotating cylinder apparatus⁽⁷⁾ and a journal bearing apparatus⁽³⁾ for a reasonable speed range in excess of the critical. Analytical attempts have also been made at explaining the circumferential modes. DiPrima⁽²³⁾ has presented a linear analysis showing rotating Couette flow to be unstable to non-axisymmetric disturbances at speeds somewhat in excess of the critical Taylor speed. More recently, DiPrima and Stuart⁽²⁴⁾ have presented a nonlinear analysis that considers both the initial growth of the Taylor vortices and the possibility of circumferential waves. Their analysis predicts the onset of the circumferential modes

with a wave number of four, and this agrees with the experimental findings of Coles⁽¹⁹⁾. To summarize, all evidence points to the appearance of torroidal vortices at the critical Taylor speed followed by the appearance of a wavy vortex regime as the speed is increased. The frequency of the "waviness" increases with rotational speed, and possibly, turbulence is superposed on the vortex structure at very high rotational speeds.

In addition to the preceding work on the stability of rotating Couette flow, there has also been considerable interest in the effect of adding an axial flow component to the flow between rotating cylinders. Since axial flow would be analogous to end-leakage in a journal bearing, this part of the literature is also relevant to the present discussion. Experimental studies with axial flows have been made by Cornish⁽²⁵⁾, Fage⁽²⁶⁾, Kaye and Elgar⁽²⁷⁾, Donnelly and Fultz⁽²⁸⁾, Snyder⁽²⁹⁾ and Astill⁽³⁰⁾. There is general agreement that a small axial flow component raises the Taylor critical speed. Quantitatively, however, the increase in critical speed seems to be a function of both the experimental apparatus and the method of determining the critical speed. Since fluid is usually introduced at one end of the experimental apparatus without initially having a rotational velocity, it is likely that the relationship between the length of the cylinders and the length required for the rotational flow component to develop plays a role in the lack of quantitative agreement. (See ref. 30.) Kaye and Elgar have reported the existence of a purely turbulent regime with sufficiently large axial flow rates. (See fig. 1). However, it is

unlikely that a journal bearing would operate with the degree of end-leakage necessary to place it in Kaye and Elgar's "Turbulent Flow" regime. The first analysis of the effect of an axial flow component was a linearized stability analysis by Goldstein⁽³¹⁾. His results predict an initial increase in Taylor critical speed with the addition of axial flow but an eventual decrease as the flow rate is increased further. Later, DiPrima⁽³²⁾ analyzed the same situation and his results predict a monotonic increase of the critical rotating speed with increasing axial flow. DiPrima concluded that Goldstein's work contained a numerical error and noted that his own predictions were in qualitative agreement with the experimental results of ref. 27 and 28 for axial Reynolds numbers less than 40. More recently, Krueger and DiPrima⁽³³⁾ repeated DiPrima's analysis in somewhat greater detail and obtained better quantitative agreement with experimental data. To summarize, the primary effect of adding a small axial flow component to the flow between rotating cylinders is merely to delay the onset of the vortex regime to higher rotational speeds. For larger axial flow rates, four possible flow regimes are reported. (See fig. 1.) However, a high-speed journal bearing would most likely operate in either the "Laminar Flow", "Laminar Flow Plus Vortices" or "Turbulent Flow Plus Vortices" regimes.

Another aspect of rotating cylinder flows in which some interest has been shown is the effect of eccentricity. Since eccentricity is responsible for the circumferential pressure profiles that support bearing loads, this part of the literature is certainly pertinent.

Unfortunately, eccentric operation has received considerably less attention than either coaxial operation with or without axial flow, and fewer definite conclusions can be drawn at this time.

Apparently the first attempt to study the flow regime between eccentric cylinders for speeds near the Taylor critical value was made by Cole⁽⁵⁾. Cole experimentally found that the vortex pattern formed much as in the concentric case for eccentricity ratios of less than 0.75 and that the onset of the secondary vortex regime took place somewhere between the Taylor critical speeds based on mean separation distance and minimum separation distance. Furthermore, the vortex spacing was approximately the same for concentric and eccentric operation, and the vortices, while being most clearly defined in the neighborhood of maximum film thickness, remained essentially parallel to each other. Cole also reported that, although back-flow in the diverging regime obscured the flow pattern at higher eccentricity ratios, he believed a transition to turbulence had taken place. It is pertinent to note that Cole performed tests with a 180° cylindrical segment, and irregular vortex patterns were observed in this case as well.

Cole's work has recently been repeated and extended by Vohr⁽³⁴⁾, who presented much of his results in motion picture form.* Vohr's results are in essential agreement with Cole's observations. For eccentricity ratios of less than 0.7 torroidal vortices appeared at a rotational speed above the critical Taylor speed for concentric operation.

* Vohr's work was carried out under NASA contract NAS w-771, and the present authors would like to express their appreciation to the Space Electric Power Office, Lewis Research Center for making the film available.

The vortices were either wavy from the outset or became wavy as the rotational speed was increased. It also appeared that the number of circumferential waves increased with speed. The vortex structure remained discernible at speeds two orders of magnitude greater than the speed at which they first appeared. However, the vortex rings appeared incomplete at the larger eccentricity ratios, disappearing completely in region of minimum film thickness. In addition to the pictures in which at least a partial vortex structure appeared, there was a surprising sequence at a large eccentricity ratio in which it appeared that turbulence had been induced by flow separation in the region of rapidly diverging flow. All except the region of minimum film thickness appeared completely disordered, and a faint vortex structure began to make its appearance as the speed of the inner cylinder was increased. Quantitative results in the form of torque data and additional studies on the effect of axial flow on eccentric operation are reported by Vohr in ref. 34. Another investigation is currently being carried out by Burton et.al.⁽³⁵⁾ in which detailed hot wire data are being obtained in apparatus that has an inner cylinder diameter of approximately six feet. Preliminary data from this investigation are available, but a complete analysis and interpretation of data has yet to be presented. The available analytical work on flow between eccentric rotating cylinders is more primitive than the experimental work. DiPrima has presented both linear⁽³⁶⁾ and non-linear⁽³⁷⁾ stability analyses which imply that an eccentric problem can be treated as a concentric problem with a superposed circumferential pressure gradient. DiPrima's

results predict a higher critical speed for the onset of the vortex regime with the addition of a circumferential pressure variation and a relationship is given between frictional torque and circumferential pressure gradient. More recently, Kulinski and Ostrach⁽³⁸⁾ investigated the effects of inertia on the pre-vortex velocity profiles for small eccentricity ratios, and their results indicate the appearance of velocity components that are 90° out of phase with the variation in film thickness as the modified Reynolds number approaches unity. Based on Vohr's findings, these additional components have little effect on the eventual appearance of vortices. However, it is quite likely that they play some role in the structure and onset of the "wavy" vortex regime. Moreover, their existence at the onset on the vortex regime raises some question about the assumptions underlying DiPrima's analyses. To summarize then, the flow between slightly eccentric cylinders is qualitatively similar to the flow between concentric cylinders. The onset of the vortex structure is definitely delayed by eccentricity. However, even the qualitative effect of eccentricity on the waviness of the vortex structure is presently unknown. A partial vortex structure exists for moderate eccentricity ratios, and direct transition from laminar to completely disordered flow only appears at high eccentricity ratios. There is also some indication that a vortex structure will appear at high eccentricity ratios if the rotational speed is sufficiently high.

This brief review of the rotating cylinder literature indicates that under most conditions the flow between rotating cylinders exhibits a secondary vortex structure. The vortex cells may have a "wavy"

interface, but this is merely a more complicated form of secondary flow. Turbulence may perhaps be superposed on the vortex structure for cylinders with small to moderate eccentricity ratios and sufficiently high speeds. Turbulence may also appear without an accompanying vortex structure if the eccentricity ratio is large enough and the rotational speed small enough. Since most turbulent journal bearing calculations include cases with small and moderate eccentricity ratios, however, it seems safest to conclude that the turbulent regime which is alluded to in ref. 11, 12, 15, etc. is one in which turbulence is superposed on a basic vortex structure. It is pertinent to point out that this flow regime may not be reached in many cases until the rotational speed is two orders of magnitude higher than the critical Taylor speed. This implies modified Reynolds numbers in excess of 100, before turbulence sets in, and even liquid metal lubricated bearings are not currently operating in this range.

TURBULENCE ASSUMPTIONS FOR HIGH-SPEED BEARINGS

It has been pointed out that most high-speed lubrication analyses assume turbulent flow and that the current trend is toward increasingly complicated turbulence descriptions. The increased complication is in the form of added details which are available from studies of turbulent flows in ducts and boundary layers; that is, the concept of a wall layer, a shear layer, etc. are being introduced along with a certain amount of empirical data. This section will be devoted to discussing whether or not this particular approach is appropriate for high-speed

lubricant flows. The discussion will essentially be separated into two parts. The first part will deal exclusively with the turbulence assumptions for journal bearing configurations, and the preceeding section on rotating cylinder flows will be of considerable relevance. Since most turbulent journal bearing analyses include the full cylindrical configuration, no large extrapolation will be involved. The second part of the discussion will deal primarily with a slider bearing configuration.

Assuming that journal bearing flows are similar to rotating cylinder flows, it follows that some type of vortex structure is inherent to the flow regime. A direct consequence of this vortex structure is that the turbulence descriptions advocated in refs. 13, 14, and 15 are inappropriate. All of these turbulence descriptions are based on an underlying assumption of similarity to fully-developed turbulence in ducts and boundary layers. A basic premise in assuming flow similarity is that the mechanism of energy transport is similar. If a vortex structure exists, as is contended herein, it will be involved to some degree in the energy transport between the main flow and the turbulent eddies. The vortex structure may indeed play a negligible role in the energy exchange and the similarity assumption may prove to give adequate results. However, this seems highly unlikely. The more detailed descriptions of the turbulence stresses then would seem to have no firmer foundation than the primitive ones and have an added disadvantage of complicating calculations of journal bearing performance parameters.

The preceeding evaluation of the turbulence assumptions was based on the existence of a vortex structure in the flow field. Since the existence of the vortices depends on a concavity of the streamlines of the flow, this condition is peculiar to journal bearing flows. Most other bearing configurations will undergo a flow transition via separation or Tollmein-Schlichting instability rather than developing Taylor-Goertler vortices. In such cases the basic assumption of turbulence might be justified. However, the concept of improving the theory of lubrication for such configurations by the addition of detailed information from turbulent duct and boundary layer studies is still open to question. Detailed measurements of turbulent flow have been made under conditions which are not met in lubrication situations. For one thing, most turbulence data for duct flows were obtained under conditions of fully-developed flow. If the slider bearing configuration in fig. 2. is considered, it is clear that the lubricant flow will not be fully developed at the ends of the pad where sudden changes in geometry exist. Flow in these regions may be compared to the entrance region of a duct, and in analogy, these "entrance regions" may extend one-hundred times the thickness of the films from each end of the pad. The entrance effects can only be neglected if the "entrance region" is negligible compared to the region of fully developed flow. Such considerations also, of course, apply in the neighborhood of lubricant supply holes, grooves and other sudden changes in geometry. Since the validity of the turbulence models proposed in refs. 13, 14 and 15 depends upon a negligible change in turbulence energy transport due to

these geometrical features, it is not clear that they offer any general improvement to lubrication theory.

GOVERNING EQUATIONS

This section is devoted to examining the validity of neglecting the inertial terms in the equations of motion for high-speed lubricant flows. For the purposes of comparison with refs. 8 through 15 the flow will be assumed fully turbulent, and the lubricant will be characterized as a Newtonian fluid with constant properties.

For simplicity, the infinite slider bearing shown in fig. 2. will be considered, and for brevity, only the continuity equation and the momentum equation in the lateral direction will be written out. The dimensional form of the complete equations are: (See fig. 2.)

Continuity

$$\frac{\partial \bar{U}}{\partial x} + \frac{\partial \bar{V}}{\partial y} = 0 \quad (1)$$

x Momentum

$$\begin{aligned} \bar{U} \frac{\partial \bar{U}}{\partial x} + \bar{V} \frac{\partial \bar{U}}{\partial y} = & - \frac{1}{\rho} \frac{\partial \bar{P}}{\partial x} + \nu \left(\frac{\partial^2 \bar{U}}{\partial x^2} + \frac{\partial^2 \bar{U}}{\partial y^2} \right) \\ & - \frac{\partial}{\partial x} (\overline{u'u'}) - \frac{\partial}{\partial y} (\overline{u'v'}) \end{aligned} \quad (2)$$

Where \bar{P} , \bar{U} and \bar{V} are the pressure and velocity components averaged in time, $\overline{u'u'}$ and $\overline{u'v'}$ are the turbulent velocity correlations and

ρ and ν are the density and kinematic viscosity of the lubricant.

By introducing the usual non-dimensionalizing parameters,

$$\begin{aligned}x^* &= x/L; & \bar{U}^* &= \bar{U}/U \\y^* &= y/\bar{h}; & \bar{V}^* &= \bar{V}/V \\ \overline{u^*u^*} &= \frac{\overline{u'u'}}{\sigma}; & \overline{u^*v^*} &= \frac{\overline{u'v'}}{\sigma} \\ \bar{P}^* &= \bar{P}/P\end{aligned}$$

and the following non-dimensional groups

$$Re^* = \frac{\bar{U}\bar{h}}{\nu} \left(\frac{\bar{h}}{L}\right); \quad \delta^* = \frac{\bar{h}}{L}; \quad \Lambda^* = \frac{\mu UL}{P\bar{h}^2},$$

these two equations can be rewritten in the following non-dimensional form,

Continuity

$$\frac{\partial \bar{U}^*}{\partial x^*} + \left(\frac{U}{V} \cdot \frac{1}{\delta^*}\right) \frac{\partial \bar{V}^*}{\partial y^*} = 0 \quad (3)$$

x Momentum

$$\begin{aligned}Re^* \left[\bar{U}^* \frac{\partial \bar{U}^*}{\partial x^*} + \left(\frac{V}{U} \cdot \frac{1}{\delta^*}\right) \bar{V}^* \frac{\partial \bar{U}^*}{\partial y^*} \right] &= - \frac{1}{\Lambda^*} \frac{\partial \bar{P}^*}{\partial x^*} + \left[\delta^{*2} \frac{\partial^2 \bar{U}^*}{\partial x^{*2}} + \right. \\ &\left. \frac{\partial^2 \bar{U}^*}{\partial y^{*2}} \right] - \left(\frac{\sigma}{U^2} \cdot \frac{Re^*}{\delta^*} \right) \left[\delta^* \frac{\partial}{\partial x^*} (\overline{u^*u^*}) + \frac{\partial}{\partial y^*} (\overline{u^*v^*}) \right] \quad (4)\end{aligned}$$

Now for lubricant film $\delta^* \sim 0(10^{-2} \text{ to } 10^{-3})$, and from continuity considerations V must be such that $(V/U \cdot \delta^*) \sim 0(1)$. In accordance

with refs. 8 through 15, Laufer's data⁽³⁹⁾ may now be introduced to scale σ . These data indicate that $(\sigma/U^2) \sim 0(10^{-2} \text{ to } 10^{-3})$.

Therefore,

$$\left(\frac{\sigma}{U^2} \cdot \frac{Re^*}{\delta^*}\right) \sim 0(Re^*)$$

In the lubrication approximation terms of order δ^* are neglected, and equation (4) reduces to

$$\begin{aligned} \frac{1}{\Lambda^*} \frac{\partial \bar{P}^*}{\partial x^*} = & \frac{\partial^2 \bar{U}^*}{\partial y^{*2}} - Re^* \left[\bar{U}^* \frac{\partial \bar{U}^*}{\partial x^*} + \left(\frac{V}{U} \cdot \frac{1}{\delta^*}\right) \bar{V}^* \frac{\partial \bar{U}^*}{\partial y^*} + \right. \\ & \left. \left(\frac{\sigma}{U^2} \cdot \frac{1}{\delta^*}\right) \frac{\partial}{\partial y^*} (\overline{u^* v^*}) \right] \end{aligned} \quad (5)$$

In classical lubrication situations σ is identically zero and $Re^* \ll 1$. Therefore, the correct approximation is

$$\frac{1}{\Lambda^*} \frac{\partial \bar{P}^*}{\partial x^*} = \frac{\partial^2 \bar{U}^*}{\partial y^{*2}} \quad (6)$$

However, in high-speed or "turbulent" situations, $Re^* \geq 0(1)$ and the term

$$Re^* \left[\bar{U}^* \frac{\partial \bar{U}^*}{\partial x^*} + \left(\frac{V}{U} \cdot \frac{1}{\delta^*}\right) \bar{V}^* \frac{\partial \bar{U}^*}{\partial y^*} + \left(\frac{\sigma}{U^2 \delta^*}\right) \frac{\partial}{\partial y^*} (\overline{u^* v^*}) \right]$$

must be retained.

The lateral momentum equation upon which existing turbulent lubrication analyses are based, however, completely neglects the inertial terms.

$$\frac{\partial \bar{P}}{\partial x} = \mu \frac{\partial^2 \bar{U}}{\partial y^2} + \frac{\partial}{\partial y} (-\rho \overline{u' u'}) \quad (7)$$

Equation (7) was first obtained in ref. 9 by an order-of-magnitude argument which only recognized the thinness of the lubricant film. The argument therein is obviously incorrect, and equation (7) is an incorrect approximation to the complete equation for turbulent flow. Therefore, the results of existing turbulent analyses are of doubtful applicability even with the acceptance of a fully-developed turbulence flow model.

CONCLUDING REMARKS

The main conclusions of the present paper are:

(1) The exact conditions under which high speed journal bearing flows can be characterized as turbulent are not clearly delineated. Moreover, turbulence, when it does exist, is most often accompanied by a large scale vortex structure.

(2) The degree to which the turbulence structure is affected by the presence of vortices, end effects and abrupt changes in film geometry is presently unknown. Therefore, the introduction of detailed turbulence models based on data from fully-developed duct flows seems to be unwarranted.

(3) Turbulent lubrication theory incorrectly neglects the inertial terms in the equations of motion for the film. Therefore, the applicability of existing analytical results is doubtful even if the proposed turbulence models are accepted.

These conclusions imply that the field of high-speed lubrication is without a viable theory, and therefore, it is premature to attempt

calculation of design information. However, some guidance in attaining the requisite theory is contained in the preceding sections. Firstly, it must be recognized that journal and slider bearing flows are fundamentally different because of the occurrence of vortices in curved flows. Therefore, treating a journal bearing as a "rolled-up" slider bearing is incorrect in the high-speed regime. Secondly, a program of basic fluid mechanical research must precede specific work in lubrication. In the case of journal bearing configurations the pre-vortex, vortex, "wavy" vortex and turbulence plus vortex regimes must be understood. This, of course, means that the quantitative effects of eccentricity must be determined with regard to the detailed structure of the regime as well as to its effect on transition. In the case of slider bearing configurations, the effects of abrupt geometrical changes and the angle of inclination of the slider on both transition and turbulence structure must also be better understood. Finally, although it has not been discussed herein, the effects of time dependent boundary motions deserve attention. It is intuitively clear that shaft orbiting, slider oscillation, etc. will effect transition. It may well be that such large scale forced disturbances will also play a significant role in determining the structure of the flow regime. Thus, the coupling between shaft and fluid film dynamics would be a more important consideration than in the classical, low modified Reynolds number theory.

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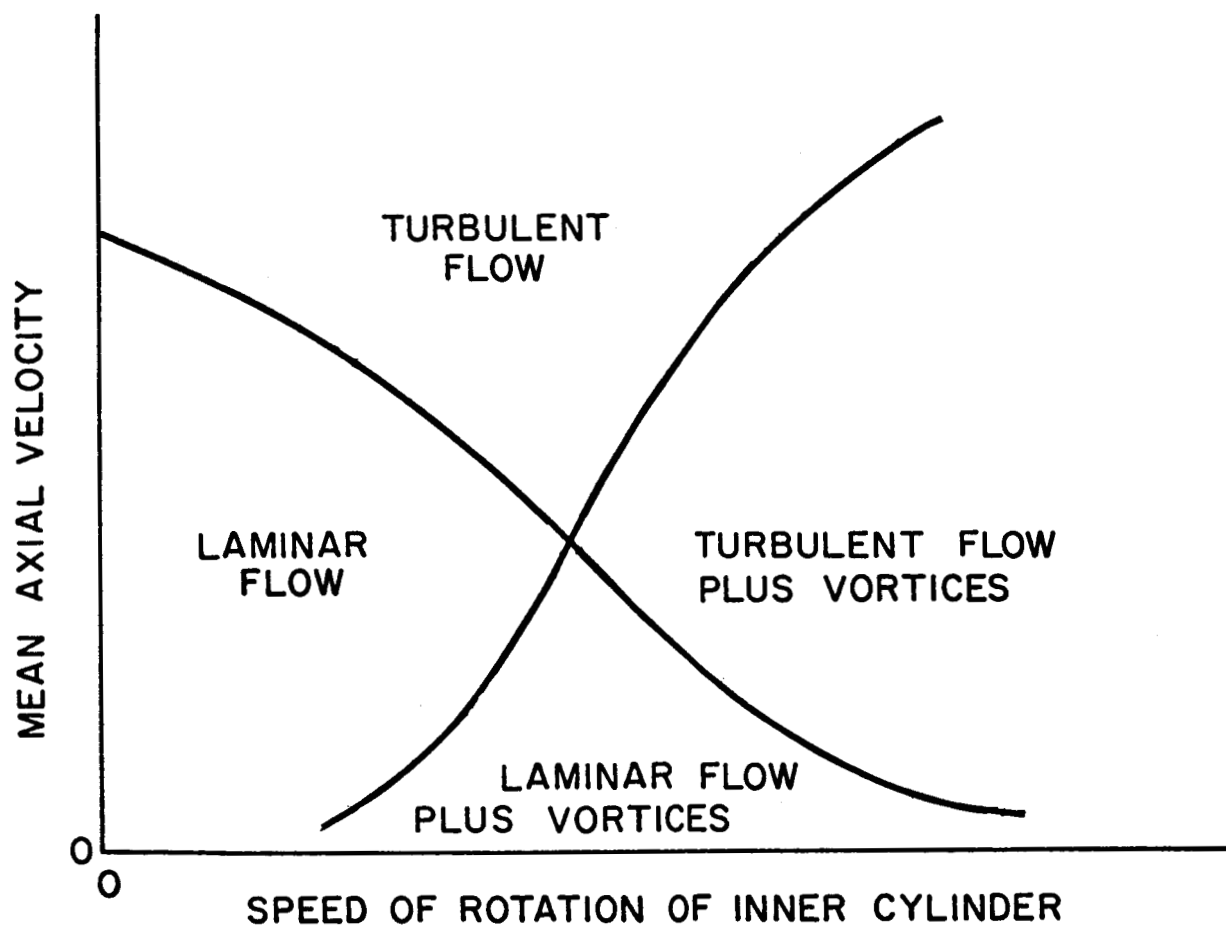


Figure 1: Schematic Representation of Different Regions of Flow in Annulus -- by Kaye and Elgar(27)

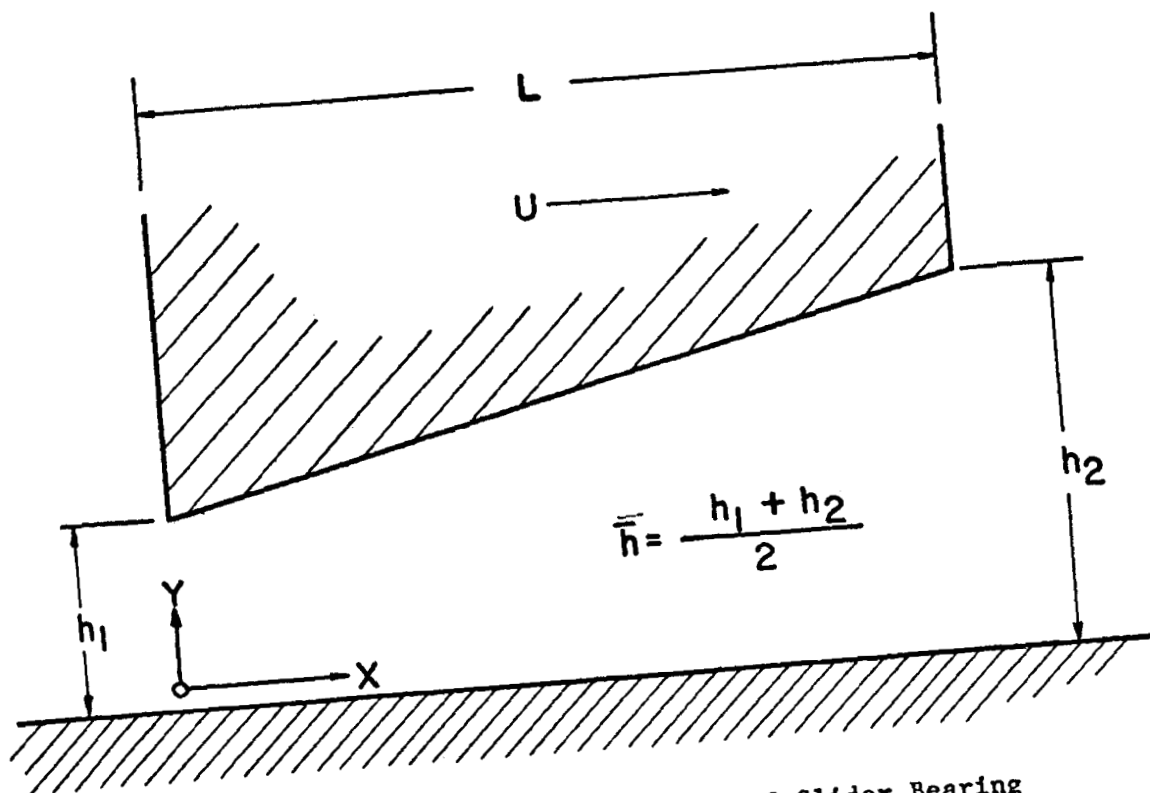


Figure 2: Schematic Representation of Slider Bearing

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